*For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!*

1. Find the minimum value of $7\cos a - 24\sin a + 9\cos b + 40\sin b + 10$.
   (A) $-56$ (B) $-66\sqrt{3}$ (C) 76 (D) 10 (E) NOTA

2. What is the phase shift of $-6\sin(4x - \frac{\pi}{8})$?
   (A) $\frac{\pi}{8}$ (B) $-\frac{\pi}{32}$ (C) $-\frac{\pi}{8}$ (D) $\frac{\pi}{2}$ (E) NOTA

3. Convert the polar equation $r = 8$ to a Cartesian equation.
   (A) $x^2 + y^2 = 64$ (B) $x^2 + y^2 = 1$ (C) $x^2 + y^2 = 8$ (D) $x + y = 8$ (E) NOTA

4. Find $\sin 18°$.
   (A) $\frac{1+\sqrt{5}}{4}$ (B) $\frac{\sqrt{6}-\sqrt{4}}{2}$ (C) $\frac{\sqrt{6}+\sqrt{4}}{2}$ (D) $\frac{-1+\sqrt{5}}{4}$ (E) NOTA

5. Find the product of the real parts of the roots of the polynomial $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$.
   (A) $\frac{1}{16}$ (B) 1 (C) $\frac{-1}{16}$ (D) $\frac{1}{8}$ (E) NOTA

6. Let vector $a = i - x^2 j - 4k$ and vector $b = 3xi - j + k$. Find the sum of all values of $x$ such that the angle between $a$ and $b$ is $\frac{\pi}{2}$
   (A) $\frac{2\sqrt{3}}{3}$ (B) $-4$ (C) 16 (D) $-3$ (E) NOTA

7. Find the centroid of a triangle with vertices $(4, 5, -3), (-5, -7, 5)$ and $(3, -5, 4)$.
   (A) $(4, -7, 4)$ (B) $(4, \frac{17}{3}, 4)$ (C) $(1, \frac{-7}{2}, 3)$ (D) $(\frac{2}{3}, \frac{-7}{3}, 2)$ (E) NOTA

8. How many petals does $r = 4\sin(6\theta)$ have?
   (A) 10 (B) 24 (C) 12 (D) 6 (E) NOTA

9. Find the projection of $v = (6\sqrt{2}, 4)$ onto $w = (2, 2\sqrt{2})$.
   (A) $(8\sqrt{2}, \frac{2}{3})$ (B) $6\sqrt{3}$ (C) $(5\sqrt{66}, \frac{10}{3})$ (D) $(15\sqrt{2}, 10\sqrt{2})$ (E) NOTA

10. Evaluate $4(\sin(15°) + \cos(15°))^4$.
    (A) 4 (B) 8 (C) 9 (D) 16 (E) NOTA

11. What is the closest integer to $\frac{2020^3}{1^2 + 2^2 + \cdots + 2020^2}$?
    (A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

12. Given that $x = \frac{\pi}{7}$, compute
$$\left(\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}\right)^2 + \left(\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}\right)^2.$$
    (A) 0 (B) 1 (C) $\pi$ (D) $-1$ (E) NOTA

13. In triangle $ABC$, $\angle A = 60°$, $\angle B = 75°$, and side length $BC$ is 4. Find the length of $AB$.
(A) $\frac{8\sqrt{3}}{3}$ (B) $\frac{4\sqrt{6}}{3}$ (C) $2\sqrt{2}$ (D) $\frac{8\sqrt{2}}{3}$ (E) NOTA

14. Eric has a hat that is bounded by $y = -5$, $y = 6$, and $y = 7 - |x|$. Find the area of Eric's hat.
(A) $\frac{231}{2}$ (B) 121 (C) 132 (D) 143 (E) NOTA

15. Tanmay and Ananya run from Rickards High School at the same time. Tanmay runs 45° east of due south at 6 miles per hour and Ananya runs due north at 2 miles per hour. What is the square of the distance between Tanmay and Ananya after 150 minutes?
(A) $250 + 75\sqrt{2}$ (B) $40 + 12\sqrt{2}$ (C) $15\sqrt{2}$ (D) $5 + 15\sqrt{2}$ (E) NOTA

16. Let $f(x) = \sin(x) + \cos(x)$. Compute $f(x) + f'(x) + f''(x) + f'''(x) + f''''(x)$.
(A) $f(x)$ (B) $f'(x)$ (C) $f''(x)$ (D) $f'''(x)$ (E) NOTA

17. Compute the absolute maximum of the function $g(x) = \frac{x}{x^2+4}$.
(A) 0 (B) 0.25 (C) 0.5 (D) 1 (E) NOTA

18. Calculate the derivative of $x^5 + 4x^2$ evaluated at $x = 0.1$.
(A) 0.8001 (B) 0.8003 (C) 0.8005 (D) 0.8007 (E) NOTA

19. Evaluate $\sin(\frac{5\pi}{12})$.
(A) $\frac{\sqrt{2}+\sqrt{6}}{4}$ (B) $\frac{\sqrt{3}+\sqrt{5}}{9}$ (C) $\frac{\sqrt{3}+\sqrt{7}}{4}$ (D) $\frac{\sqrt{5}+\sqrt{7}}{9}$ (E) NOTA

20. Compute the derivative (with respect to $x$) of the function $f(x) = 1 + x + x^2 + x^3 + \ldots$, evaluated at $x = 0.5$.
(A) 4 (B) 8 (C) 12 (D) 16 (E) NOTA

21. Let $M$ be the absolute minimum of $f(x) = \sin(x^4) + \cos(x^4)$. Let $N$ be the absolute minimum of $g(x) = \sin^4(x) + \cos^4(x)$. Compute $\left(\frac{M}{N}\right)^2$
(A) 1 (B) 2 (C) 4 (D) 8 (E) NOTA

22. Let $\theta$ be an angle such that $3\sin\theta + 4\cos\theta = 5$. Compute $4\sin\theta + 3\cos\theta$.
(A) $\frac{18}{5}$ (B) 4 (C) $\frac{24}{5}$ (D) 6 (E) NOTA

23. Evaluate the following series:
$$\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \cdots.$$
(A) $\frac{7}{4}$ (B) $\frac{15}{8}$ (C) 2 (D) $\frac{9}{4}$ (E) NOTA

24. Which of the following functions is not differentiable over all real numbers?
(A) $\cos(x^2)$ (B) $\frac{1}{x^2+1}$ (C) $\sec(1 + 0.5\sin x)$ (D) $\sec(1 + 2\sin x)$ (E) NOTA

25. Given that vector $P$ is $(2, 3, 5\sqrt{5})$ and vector $Q$ is $(4, 3\sqrt{2}, 7)$, calculate the dot product of these two vectors.
(A) $7 + 8\sqrt{3} + 28\sqrt{3}$ (B) $3 + 7\sqrt{2} + 28\sqrt{5}$ (C) $9 + 2\sqrt{3} + 35\sqrt{3}$ (D) $8 + 9\sqrt{2} + 35\sqrt{5}$ (E) NOTA

26. Given that $\sin x + \cos x = \frac{1}{5}$ and $\frac{\pi}{2} < x < \pi$, calculate $\tan x$.

(A) $\frac{3}{4}$ (B) $-\frac{3}{4}$ (C) $\frac{4}{3}$ (D) $-\frac{4}{3}$ (E) NOTA

27. Given that $i = \sqrt{-1}$, find the value of $-(1-i)^{12}$.

(A) 32 (B) 64 (C) 128 (D) 256 (E) NOTA

28. Assume that the solution to the expression $(\frac{\sqrt{3}}{4} - \frac{1}{4}i)^8$ can be expressed as $\frac{-a}{b} + \frac{c}{d}i$ where $a, b$, and $d$ are positive integers and both fractions are simplified. Compute $a - c$.

(A) $1 + \sqrt{3}$ (B) $1 - \sqrt{3}$ (C) $3 + \sqrt{3}$ (D) $3 - \sqrt{3}$ (E) NOTA

29. Find the transpose of the following matrix: $\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$

(A) 4 (B) 5 (C) 8 (D) 10 (E) NOTA

30. One of the solutions to the equation $4(x+1)^3 = 3x + 3.5$ can be expressed as $-a + \cos(b^\circ)$ where $a, b$ are positive integers and $a + b$ is minimized. Compute $a + b$. (Hint: $\cos(3x) = 4\cos^3(x) - 3\cos(x)$.)

(A) 19 (B) 20 (C) 21 (D) 22 (E) NOTA